Optimal futures trading in the presence of liquidity risk

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Abstract

A trading strategy for futures traders is proposed that is mean-variance optimal with respect to liquidity risk. It is characterized by multiple-day rollover from short to longer maturity futures contracts. Applying the strategy to six leading US grain futures markets (corn, oats, wheat, soybean, soybean meal, and soybean oil) from 2007 to 2013, the results demonstrate that considerable gains are possible over the conventional single-day rollover strategy. These benefits are consistent over the markets and over the time of the day in which trading occurs. As expected, the diversified nature of our proposed strategy means that benefits increase with liquidity risk-aversion.

Key Words: Rollover strategy, grain futures, liquidity risk.

JEL Classification Codes: C11, G11, G13.

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1 Introduction

The advent of electronic trading platforms has led to an increasing number of financial institutions employing algorithmic trading systems. A key component of these systems is *trade scheduling*, defined according to a *trade target*, that is, the number of asset units (shares or contracts) to be bought or sold during a pre-specified finite time horizon. Specifically, trade scheduling involves specifying the rate at which these asset units are traded over this period (referred to as the *trade list*) in order to optimize execution quality. Within the context of futures markets, a simple-to-implement trade scheduling procedure is proposed that solves the problem of when traders in such markets should switch from contracts that are close to maturity to deferred contracts (henceforth referred to as the *rollover strategy*). This strategy applies irrespective of whether the user has speculative, hedging, or arbitrage motives.

Trade scheduling is important as it is a significant determinant of the overall success of a trading strategy. The academic literature has recognized this importance. The vast majority of studies propose strategies that optimize the tradeoff between *pricing impact* and *timing risk*; see Almgren and Chriss (2001), He and Mamaysky (2005), Kissell and Malamut (2005), Engle and Ferstenberg (2007), Ly Vath et al. (2007), Schied and Schoeneborn (2009), Basak and Chabakauri (2010), Forsyth (2011), Lorenz and Almgren (2011), Almgren (2003, 2012), Forsyth et al (2012), and Tse et al. (2013) for a representative sample.[1] This literature is complemented by proposing a novel strategy that is designed specifically to address the rollover decision faced by users of futures markets. Our proposal shares the objectives of the extant literature, but differs in terms of the tradeoff undertaken.

Trade scheduling proposals typically adopt the following framework. A trader wishes to sell a fixed number of asset units over a finite horizon. Trade execution quality is measured by comparing the total revenue generated by selling at the *arrival price* (the price observed when the trade instruction is received) and the total revenue generated by selling these units over the horizon, with the difference referred to as the *implementation shortfall* (Perold, 1988).[2] Within this context, pricing impact is generated by assuming that the asset is subject to trading costs that increase disproportionately with the trading rate (for instance, most studies adopt a quadratic cost model). This provides the incentive to avoid fast liquidation. By contrast, timing risk represents the risk of trading at prices away from the arrival price (induced by assuming that prices evolve in a stochastic fashion). These two effects are commonly balanced such that execution quality is optimized within a mean-variance framework in which the expected implementation shortfall is minimized subject to a pre-specified implementation shortfall variance. Moreover, these moments are determined from the perspective of a single trader prior to the trading taking place.[3]

Within our setup a (futures) trader holds a number of contracts in the first month maturity contract set and wishes to schedule trades over the period prior to the first notice day (FND) such that the position is replaced by a corresponding position in the first back month contract set.[4] Applying the conventional trade scheduling approach would not be appropriate. To see this, first note that the current application involves simultaneous trading in two (near) perfectly correlated price series (that is, prices of the first month and first back month contract sets). Consequently, positive future shocks to prices will lead to falls in the implementation shortfall associated with the former contract set; however, the implementation shortfall associated with the latter contract set will rise and exactly offset the former contract set implementation shortfall. The net effect means that there is essentially no timing risk in the strategy. For this reason, a different perspective on the problem is taken.

The approach proposed is one in which liquidity (our proxy for execution quality) is optimized from the perspective of a trading desk manager who is concerned about the ex post mean and variance of liquidity over a series of trade lists (cf. the perspective of a single trader in the extant literature who is concerned about the ex ante moments of a single trade list).[5] Within our framework, the mean-variance optimal trading desk manager will typically face a tradeoff between achieving a maximum level of mean liquidity with high variance (achieved by focusing trading on a particular day prior to the FND), against a lower level of mean liquidity with lower variance (achieved by spreading trading over several days).

The mean-variance approach adopted in this paper is analogous to the approach taken in the portfolio theory literature. Consequently, we are able to borrow concepts from this well-developed literature. In particular, the efficiency of competing rollover strategies is examined using Bayesian inference based on Monte Carlo simulation; see, e.g., Kandel et al. (1995) for use of this technique within the portfolio theory literature. To anticipate some of our results, using data from six main US grain futures markets (corn, oats, wheat, soybean, soybean meal, and soybean oil), we find that rollover strategies based on trading on single days prior to the FND (as is often advocated in practise) are inefficient with respect to a mean-variance optimal rollover strategy. Moreover, for a sufficiently risk-averse trading desk manager, a naïve $1/N$ strategy that acknowledges the need to diversify trading over several days, delivers an economically meaningful improvement in performance over a rollover strategy that focuses exclusively on the maximum mean liquidity day.[6]

The remainder of this paper is organized as follows. Section 2 describes the methodologies used. Section 3 contains the application. Section 4 concludes.

2 Methodologies

In this section the problem, solution, and methodologies associated with assessing the merits of this solution are presented.

2.1 Formalizing the Problem

The futures trader framework is outlined via a set of assumptions. These are provided below.

Assumption 1. A futures trader is endowed with Λ contracts in the front month futures contract set (henceforth referred to as the *type 1 set*) and wishes to roll the position forward to the first back month futures contract set (henceforth the *type 2 set*). This is henceforth referred to as a *rollover event*.

Remark. The position can consist of a positive or negative number of contracts: thus $\Lambda \in \mathbb{R}$.

Assumption 2. To avoid physical deliver of the underlying asset the trader unwinds the position in the type 1 set prior to the FND and creates a new position in the type 2 set (this is the rollover strategy).

Remark. The trader could be a (stack) hedger, speculator, or arbitrageur. All that is relevant in our framework is that the trader requires a rollover strategy.

Assumption 3. The days on which trading occurs are given by $n = \{1, 2, \ldots, N\}$, with N representing the day immediately prior to the type 1 set FND (henceforth referred to as *event time*). Given this notation, the rollover strategy requires that Λ contracts (the trade target) are held in the type 2 set on day $N + 1$.

Assumption 4. Trading can take place on one or more days prior to the FND, with $w_{1,n}\Lambda$ ($w_{2,n}\Lambda$) representing the number of type 1 (type 2) set contracts traded on the nth trading day. Here $w_{i,n}$

is the proportional number of type i contracts traded (henceforth referred to as *weights*).

Remark. The collection of weights represent the *trade list*.

Assumption 5. Positive and negative weights are possible: thus $w_{i,n} \in \mathbb{R}\,\forall n$. However, on each day, weights exactly offset each other, that is, $w_{1,n} - w_{2,n} = 0\,\forall n$; and take the same sign in each contract set type such that $w_{1,n}$ must all be negative (positive) if $\Lambda > 0$ ($\Lambda < 0$), and $w_{2,n}$ must all be positive (negative) if $\Lambda > 0$ ($\Lambda < 0$).

Assumption 6. The trading desk manager is mean-variance efficient (MVE) with respect to the aggregate liquidity associated with the rollover strategy (our measure of aggregate execution quality), with a required mean aggregate liquidity level of μ_s.

Remark. The mean and variance are calculated over a series of rollover events.

Assumption 7. The liquidity associated with trading contract type i on day n can be decomposed as follows:

$$L_{i,n} = f(w_{i,n}, \mathit{\Lambda})\, g(v_{i,n}, \sigma_{i,n}, \mathit{\Lambda}), \tag{1}$$

where $v_{1,n}$ ($v_{2,n}$) is daily trading volume per type 1 (type 2) set traded (in dollars per contract), $\sigma_{1,n}$ ($\sigma_{2,n}$) is daily realized volatility of type 1 (type 2) set price changes (in dollars per contract), and $f(w_{i,n}, \mathit{\Lambda})$ and $g(v_{i,n}, \sigma_{i,n}, \mathit{\Lambda})$ are pre-specified functions that govern the nature of the liquidity measure.

Assumption 8. The function $f(.)$ describes the impact of the relative contribution of each day's trading to aggregate liquidity and takes the following form:

$$f(w_{i,n}, \mathit{\Lambda}) = |w_{i,n}\mathit{\Lambda}|, \tag{2}$$

such that the contribution is linear and symmetric.

Remark. Linear relations of this type are a common representation of market impact and are referred to as the *quadratic cost model*; see, e.g., Almgren and Chriss (2001). This is because the total cost of trading (if we assume that liquidity is a proportional measure of per unit trading cost) would involve $w_{i,n}^2$ terms.

Assumption 9. The function $g(.)$ specifies the measure of liquidity and is given by

$$g(v_{i,n}, \sigma_{i,n}, \Lambda) = -\ln\left(\frac{\sigma_{i,n}|\Lambda|}{v_{i,n}|\Lambda|}\right) = -\ln\left(\frac{\sigma_{i,n}}{v_{i,n}}\right), \tag{3}$$

such that a liquidity measure based on the natural log of realized volatility and trading volume is considered.

Remark. The use of this type of ratio is a common characteristic of illiquidity measures; see, e.g, Amihud (2002).

Definition The aggregate liquidity associated with the rollover strategy is given by

$$L \equiv \sum_{n=1}^{N}\sum_{i=1}^{2} L_{i,n} = \sum_{n=1}^{N}\sum_{i=1}^{2} f(w_{i,n}, \Lambda) g(v_{i,n}, \sigma_{i,n}, \Lambda), \tag{4}$$

where $f(.)$ and $g(.)$ are described above.

Under the above assumptions and definition the problem can be stated in terms of the following proposition:

Proposition 1. *The objective of the MVE trading desk manager is to minimise the variance of* *aggregate liquidity (liquidity risk) subject to constraints (a) to (c):*

$$\begin{aligned}
\underset{w}{\text{minimize}} \quad & \mathit{\Lambda}^2 \boldsymbol{w}^\top \boldsymbol{\Omega} \boldsymbol{w} \\
\text{subject to} \quad & (a)\ |\mathit{\Lambda}| \boldsymbol{w}^\top \boldsymbol{\mu} = \mu_s, \\
& (b)\ \boldsymbol{\imath}^\top \boldsymbol{w} = 1, \\
& (c)\ \boldsymbol{I}\boldsymbol{w} \geq \boldsymbol{0},
\end{aligned}$$

where

$$\boldsymbol{\mu} = \begin{pmatrix} \mathrm{E}[Y_1] \\ \vdots \\ \mathrm{E}[Y_n] \end{pmatrix}, \quad \boldsymbol{\Omega} = \begin{pmatrix} \mathrm{var}[Y_1] & \ldots & \mathrm{cov}[Y_1, Y_N] \\ \vdots & \ddots & \vdots \\ \mathrm{cov}[Y_N, Y_1] & \ldots & \mathrm{var}[Y_N] \end{pmatrix}.$$

Here $\boldsymbol{w}$ *is an* $(N \times 1)$ *vector of weights associated with the type 1 set,* $\boldsymbol{\imath}$ *is an* $(N \times 1)$ *vector of ones,* $\boldsymbol{I}$ *is an* $(N \times N)$ *identity matrix,* $\mathbf{0}$ *is an* $(N \times 1)$ *vector of zeros, and* $Y_n \equiv -(\ln(\sigma_{1,n}/v_{1,n}) + \ln(\sigma_{2,n}/v_{2,n}))$. *Without loss of generality we may assume that* $|\mathit{\Lambda}| = 1$.

Proof. Given the definition of aggregate liquidity it follows that $\mathrm{var}(L) = \mathit{\Lambda}^2 \boldsymbol{w}^\top \boldsymbol{\Omega} \boldsymbol{w}$, hence the objective function given in the proposition. Similarly, constraint (a) is obtained by setting $\mathrm{E}(L) = |\mathit{\Lambda}| \boldsymbol{w}^\top \boldsymbol{\mu}$ to the required mean aggregate liquidity level μ_s. Regarding constraints (b) and (c), we first note that the position must be completely unwound (Assumption 2), and the type 1 and 2 set weights exactly offset each day and take the same sign (Assumption 5). Furthermore, as the contribution of weights to liquidity is symmetric about zero (over n), then we can further assume that the type 1 set weights sum to unity and are all non-negatively valued without loss of generality. Finally, as the solution is invariant to scaling of the objective function we may also assume that only one contract is traded. □

The above represents a standard quadratic programming problem, with the strategy based on the solution (obtained by numerical methods) referred to as the *MVE rollover strategy*.

2.2 Analogy to portfolio theory

The MVE rollover strategy problem is similar to the classic portfolio theory problem in which a portfolio of assets is constructed to minimize the return risk associated with a required mean return. Indeed, the above proposition is directly analogous to the case in which the variance of portfolio returns is minimized subject to a short sale constraint. Consequently, the same numerical procedures used in the portfolio theory literature can be used to derive a solution to the above optimization problem.

Within the portfolio theory framework the choice revolves around how much wealth should be invested in the N assets. By contrast, our problem consists of when trades should take place over N days. This choice delivers a minimum variance of portfolio returns or of aggregate liquidity. Moreover, this choice depends on the contemporaneous correlations among the N asset returns, or the lagged correlations amongst the liquidities observed on different days in the N periods prior to the FND. In both cases, repeated observations of these returns or liquidities exist over which the sample mean and variance can be estimated.

2.3 Measuring strategy performance

We wish to examine the quality of a particular rollover strategy with respect to the above MVE rollover strategy. We take from the portfolio theory literature and consider the standard deviation reduction achieved by conducting the MVE rollover strategy with respect to the competing rollover strategy. Formally, strategy inefficiency is given by

$$\rho_s \equiv \sqrt{\frac{\boldsymbol{w}_s^\top \boldsymbol{\Omega} \boldsymbol{w}_s}{\boldsymbol{w}_*^\top \boldsymbol{\Omega} \boldsymbol{w}_*}} - 1, \tag{5}$$

where $\boldsymbol{w}_*$ denotes the MVE weights, $\boldsymbol{w}_s$ denote the weights associated with the competing rollover strategy, $\boldsymbol{w}_*^\top \boldsymbol{\mu} = \boldsymbol{w}_s^\top \boldsymbol{\mu} = \mu_s$, $\boldsymbol{\imath}^\top \boldsymbol{w}_* = \boldsymbol{\imath}^\top \boldsymbol{w}_s = 1$, and $\boldsymbol{I}\boldsymbol{w}_* \geq \boldsymbol{0}$ and $\boldsymbol{I}\boldsymbol{w}_s \geq \boldsymbol{0}$. Note that the required mean aggregate liquidity level is dictated by the mean aggregate liquidity achieved via the competing rollover strategy.

A graphical representation of the potential gains to using the MVE rollover strategy is provided in panel (a) of Figure 1. Consider five pre-selected strategies represented by the points A, B, C, D and E. All points on the mean-variance frontier between A and E can be achieved by the MVE rollover strategy, including the global minimum variance (GMV) point given by C$'$.

Insert Figure 1 here

The first feature of panel (a) to note is that the points A and E must lie on the mean-variance frontier. These points represent the maximum and minimum mean liquidity levels, respectively. As the weights are all non-negative it follows that one cannot achieve mean liquidity levels outside of these limits. For all other points a reduction in variance is possible. It is clear that A is superior to B, B is superior to C, and so on (as we have assumed that they have equal variance). However, the variance reductions do not increase over this space. Indeed, points A and E deliver the same (zero) reduction in variance.

As an alterative we assume that the variance associated with strategies that deliver a mean liquidity level below the GMV mean liquidity level (μ_m) are given by the GMV. This amounts to replacing the minimum variance frontier from C$'$ to E with the vertical line linking points C$'$ to E$'$

in panel (b). Formally, strategy inefficiency is given by

$$\rho_s \equiv \begin{cases} \sqrt{\dfrac{\boldsymbol{w}_s^\top \boldsymbol{\Omega} \boldsymbol{w}_s}{\boldsymbol{w}_*^\top \boldsymbol{\Omega} \boldsymbol{w}_*}} - 1, & \text{if } \mu_s \geq \mu_m, \\ \sqrt{\dfrac{\boldsymbol{w}_s^\top \boldsymbol{\Omega} \boldsymbol{w}_s}{\boldsymbol{w}_m^\top \boldsymbol{\Omega} \boldsymbol{w}_m}} - 1, & \text{otherwise,} \end{cases} \tag{6}$$

where $\boldsymbol{w}_m$ denotes the GMV weights, $\boldsymbol{w}_*^\top \boldsymbol{\mu} = \boldsymbol{w}_s^\top \boldsymbol{\mu} = \mu_s$, $\boldsymbol{w}_m^\top \boldsymbol{\mu} = \mu_m$, $\boldsymbol{\imath}^\top \boldsymbol{w}_* = \boldsymbol{\imath}^\top \boldsymbol{w}_s = \boldsymbol{\imath}^\top \boldsymbol{w}_m = 1$, and $\boldsymbol{I}\boldsymbol{w}_* \geq \mathbf{0}$, $\boldsymbol{I}\boldsymbol{w}_s \geq \mathbf{0}$ and $\boldsymbol{I}\boldsymbol{w}_m \geq \mathbf{0}$. See Kandel and Stambaugh (1987) for a similar measure of inefficiency within the context of portfolio efficiency.

2.4 Conducting inference

There is a portfolio theory literature that proposes methods by which one can assess whether a return portfolio is mean-variance efficient. Two primary approaches are possible: ones based on classical inference and ones based on Bayesian inference; see, e.g., Gibbons et al. (1989) and Kandel et al. (1995), respectively. The former tranche derives asymptotic tests (only), and is highly technical in the presence of portfolio weights restrictions; see, e.g., De Roon et al. (2001), for the case of portfolio efficiency testing when short sale constraints are imposed. Given that the approach proposed in the current paper is analogous to such a restriction, we follow Li et al (2003) and adopt a Bayesian approach that is able to incorporate the finite sample uncertainty into the (posterior) distribution of the performance measure described in the previous subsection.

Our observations consist of data associated with T contract pairs (type 1 and 2 sets). Let $t = \{1, \ldots, T\}$ index the rollover events over time, and $\boldsymbol{Y}_t$ be an $(N \times 1)$ vector containing the liquidity levels for day n observed for rollover event t. Assume that $\boldsymbol{Y}_t$ has a multivariate normal distribution (independently across n), with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Omega}$. We use a standard diffuse prior for this distribution

$$p(\boldsymbol{\mu}, \boldsymbol{\Omega}) \propto |\boldsymbol{\Omega}|^{-(N+1)/2}. \tag{7}$$

Given a sample of T observations, the joint posterior distribution of $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ is given by

$$p(\boldsymbol{\mu}, \boldsymbol{\Omega}|\mathbf{Y}_1, \ldots, \mathbf{Y}_T) = p(\boldsymbol{\mu}|\boldsymbol{\Omega}, \widehat{\boldsymbol{\mu}}, \mathbf{Y}_1, \ldots, \mathbf{Y}_T) \times p(\boldsymbol{\Omega}|\widehat{\boldsymbol{\Omega}}, \mathbf{Y}_1, \ldots, \mathbf{Y}_T), \tag{8}$$

where $\widehat{\boldsymbol{\mu}}$ and $\widehat{\boldsymbol{\Omega}}$ are the sample counterparts to $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$. Standard results demonstrate that the marginal posterior distribution $p(\boldsymbol{\Omega}|\widehat{\boldsymbol{\Omega}}, \mathbf{Y}_1, \ldots, \mathbf{Y}_T)$ is the inverted Wishart distribution with scale matrix $T\widehat{\Omega}$ and $T-1$ degrees of freedom, and the conditional distribution $p(\boldsymbol{\mu}|\boldsymbol{\Omega}, \widehat{\boldsymbol{\mu}}, \mathbf{Y}_1, \ldots, \mathbf{Y}_T)$ is the multivariate normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Omega}/T$.

The marginal posterior distribution of ρ_s is a complicated function of the joint posterior distribution of $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$. Consequently, an analytical derivation is not possible. Instead, Monte Carlo simulation is used to derive the empirical distribution based on the computed values of ρ_s; see Geweke (1989) for a demonstration of the accuracy of this approach. Specifically, we repeatedly draw (100,000 times) a random sample of $\boldsymbol{\mu}$ and $\boldsymbol{\Omega}$ from the above posterior distributions and compute the ρ_s value for each sample. These are then used to construct the empirical distribution of ρ_s upon which inference is conducted. We refer to this as the *Bayesian approach* in the remainder of the paper.

3 Application

In this section the foregoing methodologies are applied to a grain futures dataset.

3.1 Data

All trades in the corn, oats, wheat, soybean, soybean meal and soybean oil futures markets traded on the Chicago Mercentile Exchange (CME) over the period, January 1, 2007, to December 31, 2013, are considered. In particular, transaction prices and trading volumes associated with all type 1 and type 2 sets were obtained for each market (with ticker symbols CN, OA, WC, SY, SM, and BO, respectively) from *TickData, Inc.*.[7] These data were collected over the daytime and nighttime trading periods in which both markets are open.[8] A visual representation of how the final dataset is constructed is provided in Figure 2.

Insert Figure 2 here

The type 1 and type 2 sets are synchronized such that overlapping data over event time from 25 days before to one day before the FND (which occurs on the last business day of the month prior to the expiration month) for each rollover event in the sample are considered. Missing data (or no trading days) associated with either set type result in pairwise deletion of the observation over both sets (though this occurs very rarely). A rollover event coincides with the maturity of each contract in the dataset. Corn, oats, and wheat futures markets each have five maturity cycles per year, while the soybean futures market has seven, and the soybean meal and soybean oil futures markets each have eight of these cycles per year. With seven years of data used, this gives rise to 35 (corn, oats, and wheat), 49 (soybean), and 56 (soybean meal and soybean oil) rollover events. Moreover, as there are 25 observations per rollover event this gives 875, 1225, and 1400 daily observations, respectively.

3.2 Constructing the liquidity measure

The liquidity measure given in (3) is constructed using realized volatility and trading volume. These are based on intraday data. To concisely define these measures, some preliminary notation is required: let day n have unit length, and let the full grid of all observation points be given by $\mathcal{G} = \{n_1, \ldots, n_m\}$. Given this notation, the following realized volatility and trading volume estimators can be defined:

$$\sigma_{i,n} = \frac{1}{m} \sum_{n_j \in \mathcal{G}} |P_{n_{j,+}} - P_{n_j}|, \tag{9}$$

$$v_{i,n} = \frac{1}{m} \sum_{n_j \in \mathcal{G}} V_{n_{j,+}}, \tag{10}$$

where P_{n_j} is the price observed during the jth intraday period of day n, $n_{j,+}$ represents the next observation after n_j on $\mathcal{G}$, and $V_{n_{j,+}}$ is the trading volume observed between the n_j and $n_{j,+}$ intraday period of day n. We use five-minute frequency data to construct these measures.[9]

3.3 Estimation details

For each replication of the data within the Monte Carlo simulation, the quadratic programming problem in Proposition 1 is solved for each grain futures market using the QPROG application in GAUSS 11.0 (64-bit version). If the algorithm fails to return a converged solution then the replication is discarded and a fresh replication of the data is taken.

3.4 Summary statistics

A summary of the data is provided in Table 1. The mean realized volatility (RV), trading volume (TR), and an illiquidity (IL) measure based on the ratio of RV and TR are presented.[10]

The data are categorized according to the number of days to the FND, such that we consider data with: one to five, six to ten, eleven to fifteen, sixteen to twenty, and twenty one to twenty five days to the FND. Moreover, we further categorise the data in terms of two trading period assumptions. The first considers data from any trading period in the sample (referred to as *sample A*), and the second considers data from Monday to Friday daytime trading periods only (referred to as *sample B*).

Insert Table 1 here

The results highlight three main characteristics of the data. First, the corn futures market is the most liquid (least illiquid) grain futures market in the sample, while the oats futures market is the least liquid. Second, realized volatility, trading volume, and (more importantly) liquidity observed in sample B are higher than that observed in sample A. For this reason we make exclusive use of the former sample in the subsequent analysis. Third, there is systematic variation in liquidity over event time. Specifically, the period from six to ten trading days prior to the FND is the most liquid. This finding is consistent with the observation that contracts generally rollover during this period.[11]

The deterministic nature of liquidity can also be appreciated by the shape of the mean aggregate liquidity plots in panel (a) of Figure 3. These are based on standardizing aggregate liquidity associated with each rollover event such that they each have a zero mean and unit variance, and are plotted against event time (in days).[12] The aggregate liquidity measure for each day prior to the FND is constructed by assuming that the weight equals unity for each day in event time.[13]

Insert Figure 3 here

The plot in panel (a) of the figure demonstrates that there is a clear pattern in liquidity for each grain futures market. In particular, liquidity increases up to around eight days prior to the FND and then decreases. It also shows some variation in the pattern, with some markets exhibiting less defined patterns; see, e.g., oats futures market liquidity.

3.5 Strategy inefficiency

To assess the performance of the proposed MVE rollover strategy information relating to the distribution of the inefficiency measure ρ_s obtained using the Bayesian approach described in subsection 2.4 is provided. These inefficiencies are calculated by comparing the MVE rollover strategy with an s-day rollover strategy in which the rollover is concentrated on a single day (day s) only. The MVE rollover strategy is restricted to deliver a mean aggregate liquidity level equal to that delivered by the s-day rollover strategy. The results are provided in Table 2.

Insert Table 2 here

The results demonstrate that the s-day strategies are highly inefficient over all grain futures markets. Moreover, these inefficiencies are present over all days, though some variation is apparent. For instance, in the corn futures market, a rollover focused on one day before the FND has a mean inefficiency of 5.19 (that is, liquidity is 5.19 times higher when the MVE rollover strategy is used). By contrast, a rollover eight days before the FND has a mean inefficiency of 2.14. It can also be observed that the inefficiencies are at there minimum value around eight days prior to the FND. Indeed, the distribution of the inefficiency measure indicates that the benefit of using the MVE rollover strategy is at least zero with a posterior probability of 0.9 (or higher) around this period.

The extent of the reduction in variance that can be achieved using the MVE rollover strategy can be seen in panel (b) of Figure 3. Here the variances associated with this strategy are plotted against the time to FND. As aggregate liquidity is standardized it follows that the variance associated with the s-day rollover strategy equals unity over this space.[14] The variance levels are considerably lower than unity, with a peak occurring around eight days prior to the FND.

The variance peak coincides with the maximum mean aggregate liquidity level. Consequently, the MVE rollover strategy tends to be more concentrated (less diversified) in order to achieved this level of mean liquidity (see Figure 1 for visual support for this argument), and thus delivers a higher variance. However, some reduction in variance is possible because the location of the maximum mean aggregate liquidity day varies over the replications of the data within the simulation (which, in turn, reflects the fact that the maximum mean aggregate liquidity day varies over the sample).[15]

It is also of interest to examine the characteristics of the MVE rollover strategy in terms of the number of non-zero weights (trades on different days in which rollover occurs) required to match the mean aggregate liquidity on each day. Panels (a), (b), (c) and (d) of Figure 4 provide various information relating to these weights. Panel (a) contains the mean number of trades, panel (b) contains the mean number of small trades (defined as weights below 0.05), panel (c) contains the mean number of median trades (defined as weights above 0.05 and below 0.10), and panel (d) contains the mean number of large trades (defined as weights above 0.10).

Insert Figure 4 here

The plots reveal a number of interesting characteristics. First, the MVE rollover strategy is characterized by a diversified portfolio, with a large number of different days used to achieved the required mean aggregate liquidity level. However, the amount of trading on each day is thinly spread, with only a small number of different days in which large trades occur. Finally, the degree of diversification decreases around the maximum mean aggregate liquidity day, with the number of different days required reaching a minimum around eight days prior to the FND.

3.6 A simple diversification strategy

The results presented thus far provide evidence that the MVE rollover strategy offers clear benefits. However, from a practical perspective, a number of challenges remain. Most importantly, it is not clear how a trader would actually implement the strategy in the absence of accurate information concerning the mean and variance of liquidity. An obvious case in point here would be the inaccuracies associated with out-of-sample estimates of these moments.

The portfolio theory literature has recently proposed alternatives to the MVE approach that avoid the need for moment estimates. In particular, DeMiguel et al. (2009) demonstrate that a naïve strategy based on investing equal amounts in each asset (referred to as the $1/N$ portfolio strategy) is not outperformed by a wide range of MVE-based strategies (including one in which short sales are not permitted).

Inspired by the $1/N$ portfolio approach, we consider an analogous approach in this paper.[16] In particular, a rollover strategy in which the trader equally spreads the rollover centered on eight days prior to the FND is considered.[17] We consider strategies with weights equal to unity on the center point (referred to as a window size of one), weights equal to one third on each day around (and including) the center point (a window size of two), and so on, up to a window size of eight. The distribution of the liquidity mean and variance associated with these strategies obtained using the Bayesian approach is provided in Table 3.

Insert Table 3 here

The key feature of the results in Table 3 is that a mean-variance tradeoff is generated. When rollover occurs exclusively on the center point, a high mean and variance is obtained. As the degree of rollover diversification increases, a lower mean *and* variance is obtained. Thus, this provides a choice to the trading desk manager that they can optimize based on their risk preferences. For a manager that seeks a high mean liquidity level (low risk-averse manager), a concentrated rollover is suitable. By contrast, for a manager seeking a low variance liquidity level (high risk-averse manager), a diversified rollover is best. What is clear is that a trader cannot have both a high mean liquidity level *and* a low variance liquidity level.

3.7 Economic significance

To assess the economic significance of a strategy (say strategy $\mathcal{A}$) with respect to a competing strategy (say strategy $\mathcal{B}$), we consider the level of compensation δ ('performance fee' measured in terms of liquidity) that the user of strategy $\mathcal{A}$ must receive in order to be as well off as the user of strategy $\mathcal{B}$; see Fleming et al. (2001) for use of such a performance measure within the portfolio theory literature. As liquidity is standardized, it follows that the performance fee is given in terms of liquidity standard deviations.

Formally, a manager with the following preferences is assumed:

$$U(L) = \theta \mathrm{E}[L] + (1-\theta)\mathrm{var}[L], \quad \theta \in [0,1], \tag{11}$$

where $\mathrm{E}[L] = \boldsymbol{w}^\top \boldsymbol{\mu}$, $\mathrm{var}[L] = \boldsymbol{w}^\top \boldsymbol{\Omega} \boldsymbol{w}$, and θ measures the risk preferences of the trading desk manager. Within this context, we seek δ in

$$U(L_{\mathcal{A}} + \delta) = U(L_{\mathcal{B}}). \tag{12}$$

Using (11) and this definition we can rearrange to obtain

$$\delta^* = \mathrm{E}[L_{\mathcal{B}}] - \mathrm{E}[L_{\mathcal{A}}] + \left(\frac{1-\theta}{\theta}\right)(\mathrm{var}[L_{\mathcal{B}}] - \mathrm{var}[L_{\mathcal{A}}]), \quad \theta \in (0,1], \tag{13}$$

where δ^* represents the root of (12).

The distribution of δ^* obtained using the Bayesian approach is provided in Table 4. Here strategy $\mathcal{A}$ represents the concentrated rollover strategy (rollover exclusively eight days before the FND), and strategy $\mathcal{B}$ represents the diversified rollover strategy (with a window size of eight). We consider $\theta = \{0.1, \ldots, 0.9, 1.0\}$.

Insert Table 4 here

The results indicate that for trading desk managers with an exclusive mean focus (that is, $\theta = 1$), the mean performance fee is negative. That is, trading desk managers using strategy $\mathcal{A}$ would pay to avoid using strategy $\mathcal{B}$. This is unsurprising given the superior performance of strategy $\mathcal{A}$ in terms of mean liquidity (see Table 3). As the preference for mean switches toward a preference for variance (that is, θ falls), the performance fee increases and becomes economically significant in terms of magnitude. Importantly, the performance fees are at least positive with a posterior probability of 0.99, when θ falls below 0.5 (though this varies over the markets). Thus, for a sufficiently risk-averse trading desk manager, the diversified rollover strategy has (economic) value.

3.8 Robustness check

The analysis thus far has assumed that traders rollover their positions over the entire trading day. To examine this supposition we repeat the analysis under the assumption that trading occurs during more specific periods within the trading day. In particular, three intraday periods are considered: viz., the early trading session (8.30am to 10.00am), the mid trading session (10.01am to 12.00pm), and the late trading session (12.01pm to 1.15pm). The mean and variance associated with the MVE strategies applied to corn futures data are plotted in panel (c) and (d) of Figure 3. In addition, the distribution of the performance fee associated with corn futures obtained using the Bayesian approach is summarized in Table 5.[18]

Insert Table 5 here

The plots in Figure 3 indicate that the MVE strategy means and variances are similar across the three intraday trading sessions, and are comparable to the plots associated with the full day trading session. Moreover, the results in Table 5 indicate that performance fees are above zero with posterior probability 0.99 for θ levels below 0.5 – a result that is similar to that observed previously. Thus, the results appear robust to variation in the time at which the trader rolls their position within the trading day. Collectively, there is robust evidence in favor of the use of a diversified rollover strategy.

4 Conclusion

In this paper a mean-variance framework applicable to traders' who roll their positions in futures contracts from short to longer-dated contracts is proposed. The key innovation of this paper is the focus on aggregate liquidity risk from the perspective of a trading desk manager overseeing a series of trades. The empirical findings are summarized as follows:

1. There is a distinct inverted U-shaped pattern in liquidity in the 25-day period prior to the FND, with a peak mean liquidity level observed around eight days prior to the FND.

2. Rollover based on single days prior to the FND are likely to be inefficient with respect to the MVE strategy. One possible exception would be those strategies that rollover close to the maximum observed mean liquidity levels.

3. A naïve $1/N$ rollover strategy that evenly spreads the rollover around the maximum mean liquidity day delivers lower mean liquidity levels, but also considerably less volatility liquidity levels, in comparison to single-day rollover strategies. Indeed, for sufficiently risk-averse traders, the naïve $1/N$ rollover strategy is shown to be preferable.

These results have obvious implications for a variety of participants in futures markets. First, for a trading desk manager overseeing futures traders with rollover requirements, it is not advisable to undertake concentrated rollover on single days. Rather, a diversified rollover strategy should be adopted (providing they are reasonably risk-averse with respect to liquidity risk). Second, the results have implications for researchers and financial institutions who work in the area of index construction. For instance, the SP-GSCI is constructed under the assumption that commodity futures contracts rollover between the fifth and ninth business days of the month proceeding the expiration month. These rollover dates correspond to approximately twelve to sixteen days prior to the FND. Consequently, those seeking to track this index are likely to be exposed to too high a level of liquidity risk; see Mou (2011) and Hamilton and Wu (2014) for evidence of the performance of such strategies.

Bibliography

Almgren, R. (2003). Optimal execution with nonlinear impact functions and trading-enhanced risk. Applied Mathematical Finance, 10, 1–18.

Almgren, R. (2012). Optimal trading stochastic liquidity and volatility. SIAM Journal of Financial Mathematics 3, 163–181.

Almgren, R., & Chriss, N. (2001). Optimal execution of portfolio transactions. Journal of Risk, 3, 5–39.

Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time-series effects. Journal of Financial Markets, 5, 31–56.

Basak, S., & Chabakauri, G. (2010). Dynamic mean-variance asset allocation. Review of Financial Studies, 23, 2970–3016.

Bertsimas, D., & Lo, A. (1998). Optimal control of liquidation costs. Journal of Financial Markets, 1, 1–50.

Carchano, Ó., Medina, V., & Pardo, Á. (2014). Assessing rollover criteria for EUAs and CERs. International Journal of Economics and Financial Issues, 4, 669–676.

Carchano, Ó., & Pardo, Á. (2009). Rolling over stock index futures contracts. Journal of Futures Markets, 29, 684–694.

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naïve diversification: How inefficient is the $1/N$ portfolio strategy? Review of Financial Studies, 22, 1915–1953.

De Roon, F., Nijman, T., & Werker B. (2001). Testing for mean-variance spanning with short sales constraints and transaction costs: The case of emerging markets. Journal of Finance, 61, 721–742.

Engle, R., & Ferstenberg, R. (2007). Execution risk. Journal of Trading, 2, 10–20.

Fleming, J., Kirby, C., & Ostdiek, B. (2001). The economic value of volatility timing. Journal of Finance, 56, 329–352.

Forsyth, P. (2011). A Hamilton Jacobi Bellman approach to optimal trade execution. Applied Numerical Mathematics, 61, 241–265.

Forsyth, P., Kennedy, J., Tse, S., & Windcliff, H. (2012). Optimal trade execution: A mean quadratic variation approach. Journal of Economic Dynamics and Control, 36, 1971–1991.

Geweke, J. (1989). Bayesian inference in econometric models using Monte Carlo integration. Econometrica, 57, 1317–1339.

Gibbons, M., Ross, S., & Shanken, L. (1989). A test of the efficiency of a given portfolio. Econometrica, 57, 1121–1152.

Hamilton, J., & Wu, J. (2014). Effects of index-fund investing on commodity futures prices. NBER Working Paper No. 19892.

He, H., & Mamaysky, H. (2005). Dynamic trading policies with price impact. Journal of Economic Dynamics and Control, 29, 891–930.

Kandel, S., & Stambaugh, R. (1987). On correlations and inferences about mean-variance efficiency. Journal of Financial Economics, 18, 61–90.

Kandel, S., McCulloch, R., & Stambaugh, R. (1995). Bayesian inference and portfolio efficiency. Review of Financial Studies, 8, 1–53.

Kissell, R., & Malamut R. (2005). Understanding the profit and loss of trading algorithm. Institutional Investor, 41–49.

Li, K., Sarkar, A., & Wang, Z. (2003). Diversification benefits of emerging markets subject to portfolio constraints. Journal of Empirical Finance, 10, 57–80.

Lorenz, J., & Almgren, R. (2011). Mean-variance optimal adaptive execution. Applied Mathematical Finance, 18(5), pp. 395–422.

Ly Vath, V., Mnif, M., & Pham, H. (2007). A model of optimal portfolio selection under liquidity risk and price impact. Finance and Stochastics, 11, 51–90.

Ma, K., Mercer, M., & Walker, M. (1992). Rolling over futures contracts: A note. Journal of Futures Markets, 12, 203–217.

McAleer, M., & Medeiros, M. (2008). Realized volatility: A review. Econometric Reviews, 27, 10–45.

Mou, Y. (2011). Limits to arbitrage and commodity index investment: Front-running the Goldman roll. SSRN Working Paper available at http://ssrn.com/abstract=1716841.

Perold, A. (1988). The implementation shortfall: Paper versus reality. Journal of Portfolio Management, 14, 4–9.

Samuelson, P. (1965). Proof that properly anticipated prices fluctuate randomly. Industrial Management Review, 6, 41–49.

Schied, A., & Schoeneborn, T. (2009). Risk aversion and the dynamics of optimal liquidation strategies in illiquid markets. Finance and Stochastics, 13, 181–204.

Tse, S., Forsyth, P., Kennedy, J., & Windcliff, H. (2013). Comparison between the mean-variance optimal and the mean-quadratic-variation optimal trading strategies. Applied Mathematical Finance, 20, 415–449.

Notes

[1]The former of these reflects the increased costs of immediate trading based on liquidity considerations, while the latter risk measures the costs associated with delayed trading at prices away from those anticipated.

[2]Other benchmarks, such as those based on the volume-weighted average price, typically lead to strategies that do not deviate from the benchmark (Almgren, 2012).

[3]The literature based on this framework can be categorized in many ways. Perhaps the most obvious way is to label papers in terms of whether static or dynamic strategies are proposed. The former consists of a trade schedule that is mean-variance efficient with respect to implementation shortfall, but does not change over time to reflect changing market conditions (see, e.g., Almgren and Chriss, 2001). By contrast, dynamic strategies are also mean-variance efficient but adapt to dynamic variation in market conditions (see, e.g., Almgren, 2012).

[4]Within the context of commodity futures, the FND represents the day after which a rollover can occur to avoid taking physical deliver of the underlying commodity.

[5]There have been previous studies of the rollover decision in futures markets. Motivated by the abnormal volatility in futures prices close to their maturity (Samuelson, 1965), this literature focuses on the effect of single day rollover selection on the return series (and not liquidity). Using a variety of futures data, Carchano and Pardo (2009) and Carchano et al (2014) find that there are no significant differences in the return series over a variety of different rollover date selection criteria (cf. Ma et al., 1992).

[6]See DeMiguel et al. (2009) for empirical support for the use of the $1/N$ portfolio strategy within the portfolio theory literature.

[7]All prices pertain to CME Globex (electronic platform) transactions to reflect the dominance of this trading mechanism over the sample period.

[8]The current (circa November 2014) trading times of these markets are: Sunday to Friday, 7.00am to 7.45pm (CT); and Monday to Friday, 8.30am to 1.15pm (CT).

[9]There is a very large literature devoted to issues pertaining to realized volatility measures based on intraday data; see McAleer and Medeiros (2008) for a comprehensive review. We opt for a commonly-used measure.

[10]RV and TR are based on contract prices, that is, the quoted transaction price multiplied by the contract size. For instance, for corn, the price (given in cents per bushel) is multiplied by the 5,000 (contract size in terms of bushels).

[11]For instance, the online broker *AvaTrade* roll clients' grain futures contracts on the last Sunday prior to the

FND. Moreover, though the CME does not appear to make an explicit rollover date recommendation for grain futures investors, for equity index futures they impose a rollover dates eight business trades prior to the FND. By contrast, the Standard and Poor's-Goldman Sachs Commodity Index (SP-GSCI) and the Dow Jones-UBS Commodity Index (DJ-UBSCI) are constructed under the assumption that commodity futures contracts rollover (with uniform weights) between the fifth and ninth business days of the month proceeding the expiration month (referred to as the *Goldman roll*), and between six and tenth business days of the month proceeding the expiration month. These rollover dates correspond to approximately twelve to sixteen (SP-GSCI), and eleven to fifteen days prior to the FND. Meanwhile, data vendors such as Tickdata.com supply continuous futures prices under a number of different rollover assumptions that include rollover on the 20th calender day of the month preceding the expiration month. This date coincides with the period six to ten days prior to the FND.

[12]The use of standardized liquidity is useful on two counts. First, it provides an interpretable measure of aggregate liquidity, that is, it is measured in standard deviation terms. Second, it removes any trends in liquidity over the sample period. For these reasons, standardized aggregate liquidity is used in the remainder of the paper.

[13]The mean vector and covariance matrix associated with aggregate liquidity are smoothed. In particular, the estimated mean vector is obtained by using a Gaussian kernel smoother with bandwidth equal to $0.25T^{-0.4}$, where T represents the number of rollover events in the sample. The covariance matrix has by definition entries of unity on the diagonal. However, we smooth the off-diagonal elements (correlations) using the Gaussian kernel smoother, whereby each off-diagonal element is linearly weighted such that the elements closest to (furthest away from) the diagonal element receive the largest (smallest) weight. This ensures that the correlations taper to zero over this space.

[14]Note that the strategies are equivalent in terms of mean aggregate liquidity as this is an imposed constraint.

[15]The sample variances of the locations of the maximum mean liquidity days are 2.138, 3.474, 1.529, 1.366, 1.483, 1.641 days, for the corn, oats, wheat, soybean, soybean meal, and soybean oil futures markets, respectively.

[16]Within the trade scheduling literature, this is commonly referred to as the naïve strategy (Bertsimas and Lo, 1998). Investment houses often employ such naïve strategies. For instance, the Goldman roll assumes an equally-weighted roll centered on the seventh business day of the month proceeding the first month contract maturity.

[17]We select this center point as it represents the maximum mean liquidity day.

[18]Results pertaining to the other grain futures markets give similar results. These are available on request.

Table 1 – Summary Information

		Measure (sample A)			Measure (sample B)		
Days to first notice day	Observations (days)	RV	TV	IL	RV	TV	IL
(a) Corn (CN)							
1 to 5	875	56.65	18.35	1.63	166.37	100.16	0.57
6 to 10		57.47	20.07	1.15	171.46	107.56	0.50
11 to 15		59.36	20.90	1.48	181.31	116.47	0.59
16 to 20		58.05	21.47	1.57	164.96	107.31	0.67
21 to 25		57.27	21.22	1.83	161.40	103.61	0.76
(b) Oats (OA)							
1 to 5	875	29.84	0.08	302.41	119.42	0.39	117.31
6 to 10		25.04	0.07	226.78	106.68	0.37	105.21
11 to 15		28.28	0.08	197.85	111.02	0.39	112.28
16 to 20		26.89	0.08	255.51	105.38	0.40	110.15
21 to 25		26.18	0.08	373.06	97.56	0.36	204.93
(c) Wheat (WC)							
1 to 5	875	88.23	9.92	5.85	266.46	51.91	2.26
6 to 10		77.25	9.97	3.20	248.58	53.43	1.27
11 to 15		84.15	10.96	3.66	270.67	58.99	1.90
16 to 20		83.22	10.88	4.24	257.76	56.71	1.92
21 to 25		83.09	11.31	5.45	248.41	51.55	2.83
(d) Soybean (SY)							
1 to 5	1225	105.05	23.80	3.71	296.95	122.61	1.54
6 to 10		106.98	26.90	2.65	301.50	132.11	1.34
11 to 15		107.42	26.59	21.60	319.02	143.51	1.65
16 to 20		108.46	28.07	4.06	297.18	134.07	1.97
21 to 25		101.78	27.04	4.49	295.96	130.08	2.45
(e) Soybean Meal (SM)							
1 to 5	1400	25.77	2.02	9.38	81.21	10.61	4.01
6 to 10		26.90	2.20	7.78	84.60	11.43	3.48
11 to 15		27.37	2.29	9.66	88.10	11.85	4.26
16 to 20		26.27	2.16	9.77	81.69	10.98	4.83
21 to 25		25.35	2.15	9.72	79.12	10.41	5.86
(f) Soybean Oil (BO)							
1 to 5	1400	1.78	0.21	6.42	5.14	1.06	2.50
6 to 10		1.86	0.24	4.98	5.22	1.19	2.07
11 to 15		1.93	0.25	5.69	5.33	1.20	2.21
16 to 20		1.87	0.24	6.18	5.10	1.14	2.57
21 to 25		1.78	0.24	6.31	5.07	1.08	3.72

Notes: This table contains the mean levels of realized volatility (RV), trading volume (TV), and illiquidity (IL, given by the ratio of RV over TV) associated with each grain futures market, for various five-day periods prior to the FND, and for two samples. Sample A corresponds to data observed over all trading periods, while sample B corresponds to Monday to Friday daytime trading periods only. For presentation reasons, the realized volatility data are multiplied by 10, the trading volume data are divided by 100000, and the illiquidity data are multiplied by 100000.

Table 2 – MVE strategy benefits

	Statistic							
Strategy	MN	SD	IQR	50%	75%	90%	95%	99%
(a) Corn (CN)								
1 (day(s) to first notice day)	5.19	1.85	2.21	4.92	3.93	3.18	2.76	1.84
2	4.95	1.90	2.26	4.71	3.69	2.86	2.36	1.22
3	4.46	2.02	2.45	4.26	3.12	2.12	1.53	0.20
4	3.68	2.16	2.75	3.44	2.16	1.15	0.59	0.00
5	2.87	2.18	2.80	2.48	1.27	0.32	0.00	0.00
6	2.37	2.12	2.68	1.90	0.78	0.00	0.00	0.00
7	2.18	2.08	2.60	1.67	0.61	0.00	0.00	0.00
8	2.14	2.08	2.56	1.62	0.56	0.00	0.00	0.00
9	2.20	2.09	2.60	1.70	0.62	0.00	0.00	0.00
10	2.44	2.13	2.70	1.99	0.85	0.00	0.00	0.00
11	3.03	2.19	2.83	2.68	1.43	0.51	0.00	0.00
12	3.81	2.14	2.70	3.60	2.32	1.30	0.72	0.00
13	4.46	2.04	2.45	4.25	3.11	2.12	1.52	0.00
14	4.77	1.93	2.29	4.56	3.51	2.62	2.07	0.81
15	4.88	1.91	2.26	4.66	3.63	2.78	2.24	1.04
16	4.95	1.91	2.26	4.72	3.69	2.86	2.36	1.10
20	5.35	1.82	2.19	5.07	4.10	3.38	2.99	2.30
24	5.51	1.88	2.25	5.20	4.20	3.48	3.11	2.47
(b) Oats (OA)								
1	6.91	2.41	2.82	6.59	5.33	4.34	3.72	2.00
2	6.81	2.46	2.85	6.53	5.23	4.16	3.44	1.48
3	6.52	2.56	2.96	6.30	4.93	3.66	2.65	0.77
4	5.87	2.80	3.37	5.77	4.11	2.28	1.31	0.00
5	4.81	3.09	4.35	4.67	2.40	0.92	0.15	0.00
6	3.66	3.15	4.61	2.99	1.07	0.00	0.00	0.00
7	3.05	3.09	4.33	2.10	0.52	0.00	0.00	0.00
8	3.34	3.14	4.48	2.51	0.78	0.00	0.00	0.00
9	4.30	3.15	4.61	3.97	1.74	0.45	0.00	0.00
10	5.33	2.96	3.87	5.26	3.24	1.47	0.69	0.00
11	5.98	2.76	3.27	5.87	4.28	2.48	1.47	0.00
12	6.21	2.69	3.13	6.06	4.58	2.97	1.84	0.00
13	6.18	2.71	3.14	6.04	4.54	2.91	1.79	0.00
14	5.95	2.79	3.28	5.85	4.25	2.41	1.36	0.00
15	5.67	2.91	3.58	5.61	3.79	1.88	0.96	0.00
16	5.47	2.94	3.73	5.43	3.50	1.59	0.72	0.00
20	4.91	3.09	4.32	4.83	2.53	0.91	0.00	0.00
24	6.77	2.49	2.89	6.50	5.19	4.08	3.28	1.18

Table 2 – MVE strategy benefits (cont.)

	Statistic							
Strategy	MN	SD	IQR	50%	75%	90%	95%	99%
(c) Wheat (WC)								
1 (day(s) to first notice day)	3.98	1.44	1.73	3.73	2.97	2.41	2.13	1.64
2	3.97	1.43	1.71	3.73	2.98	2.41	2.12	1.64
3	3.87	1.39	1.66	3.65	2.91	2.36	2.07	1.60
4	3.40	1.38	1.65	3.18	2.45	1.90	1.60	1.10
5	2.50	1.38	1.63	2.28	1.56	1.03	0.71	0.00
6	1.70	1.33	1.55	1.47	0.79	0.14	0.00	0.00
7	1.25	1.26	1.54	0.98	0.28	0.00	0.00	0.00
8	1.13	1.23	1.64	0.84	0.03	0.00	0.00	0.00
9	1.26	1.27	1.55	0.98	0.28	0.00	0.00	0.00
10	1.53	1.31	1.52	1.29	0.62	0.00	0.00	0.00
11	1.90	1.36	1.57	1.66	0.98	0.41	0.00	0.00
12	2.27	1.38	1.61	2.05	1.35	0.80	0.47	0.00
13	2.66	1.40	1.64	2.43	1.71	1.17	0.86	0.00
14	2.95	1.39	1.65	2.73	2.00	1.45	1.14	0.46
15	3.18	1.39	1.64	2.96	2.24	1.69	1.39	0.78
16	3.39	1.38	1.66	3.17	2.44	1.89	1.59	1.04
20	3.90	1.39	1.67	3.68	2.94	2.39	2.10	1.63
24	3.99	1.44	1.73	3.76	2.99	2.42	2.13	1.66
(d) Soybean (SY)								
1	3.95	0.91	1.17	3.85	3.31	2.88	2.64	2.23
2	3.92	0.90	1.16	3.82	3.29	2.86	2.63	2.22
3	3.69	0.93	1.19	3.60	3.05	2.59	2.33	1.84
4	2.92	1.05	1.38	2.83	2.18	1.66	1.36	0.83
5	1.85	1.04	1.30	1.72	1.13	0.67	0.34	0.00
6	1.07	0.91	1.16	0.93	0.38	0.00	0.00	0.00
7	0.77	0.84	1.20	0.60	0.00	0.00	0.00	0.00
8	0.90	0.87	1.35	0.75	0.00	0.00	0.00	0.00
9	1.32	0.96	1.18	1.19	0.66	0.00	0.00	0.00
10	1.89	1.04	1.30	1.76	1.18	0.72	0.40	0.00
11	2.43	1.07	1.39	2.32	1.68	1.18	0.91	0.00
12	2.83	1.06	1.39	2.73	2.08	1.56	1.28	0.72
13	3.08	1.04	1.36	3.00	2.36	1.84	1.53	0.97
14	3.30	1.01	1.30	3.23	2.61	2.08	1.78	1.23
15	3.50	0.97	1.25	3.43	2.83	2.34	2.05	1.52
16	3.66	0.94	1.21	3.58	3.01	2.54	2.28	1.77
20	3.95	0.90	1.17	3.85	3.31	2.88	2.65	2.24
24	3.96	0.91	1.18	3.86	3.32	2.88	2.65	2.25

Table 2 – MVE strategy benefits (cont.)

Strategy	MN	SD	IQR	50%	75%	90%	95%	99%
	Statistic							
(e) Soybean Meal (SM)								
1 (day(s) to first notice day)	10.91	2.34	2.88	10.86	9.46	8.15	7.20	4.95
2	10.23	2.69	3.30	10.32	8.64	6.78	5.52	3.32
3	8.75	3.19	4.46	8.89	6.51	4.44	3.38	1.71
4	6.20	3.38	4.87	5.77	3.58	2.14	1.46	0.00
5	3.55	2.85	3.42	2.88	1.51	0.51	0.00	0.00
6	2.05	2.24	2.58	1.47	0.34	0.00	0.00	0.00
7	1.71	2.07	2.44	1.14	0.00	0.00	0.00	0.00
8	2.23	2.31	2.58	1.65	0.56	0.00	0.00	0.00
9	3.37	2.79	3.30	2.71	1.39	0.32	0.00	0.00
10	4.88	3.22	4.26	4.26	2.47	1.32	0.68	0.00
11	6.53	3.41	4.96	6.17	3.89	2.37	1.62	0.00
12	8.06	3.34	4.83	8.06	5.59	3.66	2.70	1.15
13	9.37	3.04	4.07	9.55	7.37	5.26	4.09	2.19
14	10.30	2.66	3.25	10.37	8.72	6.93	5.66	3.41
15	10.85	2.37	2.93	10.80	9.39	8.03	7.05	4.70
16	11.12	2.22	2.79	11.01	9.69	8.52	7.77	5.80
20	11.54	2.04	2.67	11.34	10.10	9.11	8.55	7.62
24	11.56	2.04	2.68	11.37	10.11	9.12	8.57	7.63
(f) Soybean Oil (BO)								
1	6.42	1.21	1.56	6.30	5.58	4.99	4.66	4.09
2	6.40	1.20	1.57	6.29	5.55	4.96	4.64	4.07
3	6.21	1.24	1.59	6.12	5.37	4.73	4.36	3.57
4	5.31	1.58	2.08	5.31	4.26	3.26	2.69	1.67
5	3.45	1.77	2.41	3.26	2.15	1.34	0.90	0.00
6	1.95	1.55	1.93	1.69	0.85	0.00	0.00	0.00
7	1.33	1.36	2.00	1.05	0.00	0.00	0.00	0.00
8	1.39	1.37	1.94	1.11	0.12	0.00	0.00	0.00
9	1.78	1.50	1.85	1.50	0.70	0.00	0.00	0.00
10	2.31	1.63	2.06	2.04	1.15	0.39	0.00	0.00
11	2.88	1.73	2.25	2.64	1.64	0.88	0.35	0.00
12	3.52	1.79	2.43	3.34	2.22	1.39	0.93	0.00
13	4.22	1.79	2.48	4.13	2.94	1.99	1.47	0.00
14	4.89	1.69	2.28	4.88	3.73	2.72	2.16	1.13
15	5.38	1.57	2.04	5.39	4.36	3.39	2.79	1.69
16	5.69	1.47	1.85	5.68	4.76	3.88	3.30	2.19
20	6.37	1.20	1.56	6.26	5.53	4.93	4.60	4.02
24	6.42	1.21	1.57	6.31	5.57	4.98	4.66	4.10

Notes: This table contains information relating to the distribution of the inefficiency measure ρ_s obtained using the Bayesian approach. Specifically, the mean (MN), standard deviation (SD), interquartile range (IR), and 50, 75, 90, 95, and 99 percentiles are provided. These inefficiencies are based on a comparison of the MVE rollover strategy with a s-day rollover strategy in which the rollover takes place on one day (day s) only.

Table 3 – Diversification strategy benefits

		Statistic							
Strategy	Moment	MN	SD	IQR	50%	75%	90%	95%	99%
(a) Corn (CN)									
1 (window size)	Mean	0.31	0.17	0.21	0.31	0.20	0.10	0.04	−0.11
2		0.31	0.12	0.14	0.31	0.24	0.17	0.12	0.02
3		0.30	0.10	0.12	0.30	0.24	0.18	0.14	0.06
4		0.28	0.09	0.11	0.28	0.23	0.17	0.14	0.07
5		0.25	0.08	0.10	0.25	0.20	0.16	0.13	0.06
6		0.22	0.07	0.09	0.22	0.17	0.13	0.10	0.04
7		0.18	0.07	0.08	0.18	0.14	0.10	0.08	0.02
8		0.15	0.06	0.08	0.15	0.11	0.08	0.05	0.00
1	Variance	1.00	0.58	0.55	0.86	0.64	0.50	0.43	0.34
2		0.47	0.27	0.25	0.40	0.30	0.23	0.20	0.16
3		0.33	0.19	0.18	0.29	0.21	0.17	0.15	0.11
4		0.26	0.15	0.14	0.22	0.17	0.13	0.11	0.09
5		0.21	0.12	0.12	0.18	0.14	0.11	0.09	0.07
6		0.18	0.10	0.10	0.15	0.11	0.09	0.08	0.06
7		0.15	0.09	0.08	0.13	0.10	0.08	0.07	0.05
8		0.13	0.08	0.07	0.11	0.09	0.07	0.06	0.05
(b) Oats (OA)									
1	Mean	0.17	0.17	0.21	0.18	0.07	−0.03	−0.10	−0.24
2		0.16	0.11	0.14	0.16	0.09	0.02	−0.03	−0.12
3		0.13	0.10	0.12	0.13	0.07	0.02	−0.02	−0.10
4		0.10	0.08	0.11	0.10	0.05	0.00	−0.03	−0.11
5		0.07	0.08	0.09	0.07	0.03	−0.02	−0.05	−0.11
6		0.05	0.07	0.08	0.05	0.01	−0.03	−0.06	−0.12
7		0.03	0.06	0.08	0.03	−0.01	−0.05	−0.07	−0.12
8		0.01	0.06	0.07	0.01	−0.02	−0.06	−0.08	−0.13
1	Variance	1.00	0.57	0.55	0.85	0.64	0.50	0.44	0.35
2		0.45	0.26	0.25	0.39	0.29	0.23	0.20	0.16
3		0.32	0.19	0.18	0.28	0.21	0.16	0.14	0.11
4		0.25	0.14	0.14	0.21	0.16	0.13	0.11	0.09
5		0.20	0.11	0.11	0.17	0.13	0.10	0.09	0.07
6		0.16	0.09	0.09	0.14	0.10	0.08	0.07	0.06
7		0.13	0.07	0.07	0.11	0.08	0.07	0.06	0.05
8		0.11	0.06	0.06	0.09	0.07	0.05	0.05	0.04
(c) Wheat (WC)									
1	Mean	0.51	0.17	0.21	0.51	0.41	0.31	0.24	0.10
2		0.50	0.12	0.15	0.50	0.43	0.35	0.31	0.21
3		0.48	0.11	0.13	0.48	0.41	0.35	0.30	0.22
4		0.44	0.10	0.12	0.44	0.38	0.32	0.28	0.20
5		0.39	0.09	0.12	0.39	0.33	0.28	0.24	0.16
6		0.33	0.09	0.11	0.33	0.28	0.22	0.19	0.12
7		0.27	0.08	0.10	0.27	0.22	0.17	0.14	0.07
8		0.20	0.08	0.10	0.20	0.16	0.11	0.08	0.01
1	Variance	1.00	0.57	0.55	0.86	0.64	0.50	0.44	0.34
2		0.50	0.29	0.27	0.43	0.32	0.25	0.22	0.17
3		0.40	0.23	0.22	0.34	0.25	0.20	0.17	0.14
4		0.34	0.20	0.19	0.29	0.22	0.17	0.15	0.12
5		0.30	0.17	0.16	0.25	0.19	0.15	0.13	0.10
6		0.26	0.15	0.14	0.22	0.16	0.13	0.11	0.09
7		0.23	0.13	0.13	0.20	0.15	0.11	0.10	0.08
8		0.21	0.12	0.11	0.18	0.13	0.10	0.09	0.07

Table 3 – Diversification strategy benefits (cont.)

Strategy	Moment	Statistic							
		MN	SD	IQR	50%	75%	90%	95%	99%
(d) Soybean (SY)									
1 (window size)	Mean	0.53	0.14	0.19	0.53	0.44	0.35	0.30	0.19
2		0.52	0.10	0.13	0.52	0.45	0.39	0.35	0.28
3		0.49	0.09	0.11	0.49	0.43	0.38	0.34	0.28
4		0.44	0.08	0.10	0.44	0.39	0.34	0.31	0.26
5		0.39	0.07	0.09	0.39	0.35	0.30	0.28	0.22
6		0.34	0.06	0.08	0.34	0.30	0.26	0.23	0.18
7		0.28	0.06	0.08	0.28	0.24	0.21	0.18	0.14
8		0.23	0.05	0.07	0.23	0.19	0.16	0.14	0.10
1	Variance	1.00	0.32	0.38	0.94	0.78	0.66	0.61	0.51
2		0.50	0.16	0.19	0.47	0.39	0.33	0.30	0.26
3		0.38	0.12	0.14	0.35	0.29	0.25	0.23	0.19
4		0.30	0.10	0.11	0.29	0.24	0.20	0.18	0.16
5		0.25	0.08	0.09	0.23	0.19	0.16	0.15	0.13
6		0.21	0.06	0.08	0.19	0.16	0.14	0.12	0.11
7		0.17	0.05	0.06	0.16	0.13	0.11	0.10	0.09
8		0.15	0.05	0.06	0.14	0.12	0.10	0.09	0.08
(e) Soybean Meal (SM)									
1	Mean	0.46	0.13	0.18	0.46	0.37	0.29	0.24	0.14
2		0.45	0.08	0.11	0.45	0.39	0.35	0.31	0.26
3		0.43	0.06	0.08	0.43	0.39	0.35	0.32	0.28
4		0.40	0.05	0.07	0.40	0.36	0.33	0.31	0.27
5		0.36	0.04	0.06	0.36	0.33	0.30	0.29	0.26
6		0.31	0.03	0.05	0.31	0.29	0.27	0.26	0.23
7		0.27	0.03	0.04	0.27	0.25	0.24	0.23	0.20
8		0.24	0.03	0.03	0.24	0.22	0.20	0.19	0.17
1	Variance	1.00	0.27	0.33	0.95	0.81	0.70	0.64	0.56
2		0.38	0.10	0.12	0.36	0.30	0.26	0.24	0.21
3		0.23	0.06	0.08	0.22	0.18	0.16	0.15	0.13
4		0.15	0.04	0.05	0.14	0.12	0.10	0.10	0.08
5		0.10	0.03	0.03	0.09	0.08	0.07	0.06	0.06
6		0.07	0.02	0.02	0.07	0.06	0.05	0.04	0.04
7		0.05	0.01	0.02	0.05	0.04	0.03	0.03	0.03
8		0.04	0.01	0.01	0.04	0.03	0.03	0.02	0.02
(f) Soybean Oil (BO)									
1	Mean	0.47	0.13	0.18	0.47	0.38	0.30	0.25	0.15
2		0.45	0.09	0.12	0.45	0.40	0.34	0.31	0.25
3		0.43	0.07	0.10	0.43	0.38	0.34	0.31	0.26
4		0.40	0.06	0.08	0.40	0.35	0.31	0.29	0.24
5		0.35	0.06	0.07	0.35	0.31	0.28	0.26	0.22
6		0.30	0.05	0.06	0.30	0.27	0.24	0.22	0.18
7		0.25	0.04	0.06	0.25	0.22	0.20	0.18	0.15
8		0.20	0.04	0.05	0.20	0.18	0.15	0.14	0.11
1	Variance	1.00	0.27	0.34	0.96	0.81	0.70	0.64	0.56
2		0.43	0.12	0.14	0.41	0.35	0.30	0.28	0.24
3		0.30	0.08	0.10	0.29	0.24	0.21	0.20	0.17
4		0.23	0.06	0.08	0.22	0.19	0.16	0.15	0.13
5		0.17	0.05	0.06	0.17	0.14	0.12	0.11	0.10
6		0.13	0.04	0.04	0.13	0.11	0.09	0.09	0.07
7		0.10	0.03	0.03	0.10	0.08	0.07	0.07	0.06
8		0.08	0.02	0.03	0.08	0.07	0.06	0.05	0.05

Notes: This table contains information relating to the distribution of the liquidity mean and variance obtained using the Bayesian approach. Specifically, the mean (MN), standard deviation (SD), interquartile range (IR), and 50, 75, 90, 95, and 99 percentiles are provided. The liquidity mean and variance are based on the $1/N$ rollover strategy (with a window size of eight).

Table 4 – Performance fees

	Statistic							
Preference $(1-\theta)$	MN	SD	IQR	1%	5%	50%	95%	99%
(a) Corn (CN)								
0.0 (mean-focused)	−0.16	0.15	0.19	0.22	0.09	−0.16	−0.41	−0.54
0.1	−0.06	0.17	0.20	0.40	0.22	−0.07	−0.31	−0.42
0.2	0.06	0.21	0.24	0.71	0.42	0.03	−0.22	−0.31
0.3	0.21	0.29	0.30	1.21	0.72	0.16	−0.12	−0.22
0.4	0.42	0.40	0.41	1.85	1.14	0.34	−0.02	−0.14
0.5	0.71	0.59	0.57	2.80	1.74	0.58	0.10	−0.04
0.6	1.14	0.85	0.83	4.26	2.66	0.94	0.27	0.09
0.7	1.86	1.32	1.28	6.63	4.24	1.54	0.54	0.27
0.8	3.29	2.26	2.15	11.39	7.25	2.75	1.07	0.62
0.9	7.65	5.03	4.81	26.24	16.67	6.41	2.64	1.66
1.0 (variance-focused)	∞							
(b) Oats (OA)								
0.0	−0.16	0.15	0.19	0.22	0.09	−0.16	−0.41	−0.54
0.1	−0.06	0.17	0.20	0.41	0.22	−0.07	−0.31	−0.42
0.2	0.06	0.21	0.24	0.73	0.43	0.03	−0.22	−0.31
0.3	0.22	0.28	0.30	1.19	0.73	0.17	−0.12	−0.22
0.4	0.43	0.41	0.41	1.88	1.15	0.35	−0.01	−0.12
0.5	0.73	0.58	0.57	2.85	1.77	0.60	0.12	−0.01
0.6	1.17	0.86	0.83	4.32	2.72	0.97	0.30	0.13
0.7	1.92	1.33	1.28	6.72	4.27	1.60	0.60	0.34
0.8	3.41	2.30	2.16	11.81	7.47	2.85	1.17	0.74
0.9	7.83	5.08	4.85	26.83	16.89	6.59	2.84	1.91
(c) Wheat (WC)								
0.0	−0.31	0.14	0.18	0.04	−0.08	−0.31	−0.54	−0.66
0.1	−0.22	0.16	0.19	0.22	0.04	−0.23	−0.46	−0.56
0.2	−0.11	0.20	0.22	0.52	0.23	−0.14	−0.37	−0.47
0.3	0.03	0.28	0.29	0.98	0.51	−0.02	−0.30	−0.40
0.4	0.22	0.39	0.39	1.60	0.91	0.14	−0.21	−0.33
0.5	0.48	0.57	0.55	2.53	1.49	0.36	−0.11	−0.27
0.6	0.88	0.84	0.80	3.91	2.36	0.69	0.02	−0.18
0.7	1.55	1.28	1.23	6.21	3.83	1.24	0.25	−0.08
0.8	2.88	2.18	2.11	10.91	6.74	2.36	0.68	0.15
0.9	6.84	4.99	4.66	24.88	15.60	5.66	1.93	0.71
(d) Soybean (SY)								
0.0	−0.30	0.13	0.17	0.00	−0.10	−0.30	−0.51	−0.61
0.1	−0.21	0.13	0.17	0.12	0.01	−0.21	−0.42	−0.51
0.2	−0.09	0.15	0.19	0.31	0.17	−0.10	−0.32	−0.40
0.3	0.06	0.18	0.23	0.59	0.39	0.04	−0.20	−0.29
0.4	0.26	0.24	0.29	1.00	0.70	0.23	−0.06	−0.16
0.5	0.55	0.33	0.40	1.60	1.16	0.50	0.11	−0.01
0.6	0.98	0.48	0.58	2.52	1.86	0.90	0.36	0.19
0.7	1.68	0.73	0.87	4.01	3.03	1.56	0.75	0.52
0.8	3.10	1.24	1.48	7.09	5.40	2.88	1.53	1.14
0.9	7.36	2.76	3.28	16.41	12.52	6.87	3.86	3.01

Table 4 – Performance fees (cont.)

	Statistic							
Preference $(1-\theta)$	MN	SD	IQR	1%	5%	50%	95%	99%
(e) Soybean Meal (SM)								
0.0 (mean-focused)	−0.22	0.13	0.18	0.09	−0.01	−0.22	−0.44	−0.54
0.1	−0.12	0.13	0.18	0.22	0.11	−0.12	−0.33	−0.42
0.2	0.02	0.15	0.19	0.41	0.27	0.01	−0.21	−0.30
0.3	0.19	0.18	0.23	0.69	0.50	0.17	−0.07	−0.16
0.4	0.42	0.22	0.28	1.09	0.82	0.39	0.10	0.01
0.5	0.74	0.30	0.38	1.65	1.29	0.70	0.33	0.21
0.6	1.22	0.43	0.53	2.54	2.01	1.16	0.65	0.49
0.7	2.02	0.65	0.80	4.01	3.22	1.92	1.17	0.94
0.8	3.63	1.10	1.35	7.05	5.66	3.45	2.20	1.83
0.9	8.45	2.46	3.00	16.17	13.02	8.05	5.25	4.41
1.0 (variance-focused)	∞							
(f) Soybean Oil (BO)								
0.0	−0.26	0.12	0.16	0.03	−0.06	−0.26	−0.47	−0.56
0.1	−0.16	0.13	0.17	0.16	0.05	−0.16	−0.37	−0.45
0.2	−0.04	0.14	0.18	0.34	0.21	−0.04	−0.25	−0.34
0.3	0.13	0.17	0.22	0.61	0.43	0.11	−0.12	−0.21
0.4	0.35	0.22	0.28	0.98	0.74	0.32	0.04	−0.05
0.5	0.65	0.30	0.37	1.56	1.20	0.61	0.25	0.13
0.6	1.11	0.42	0.52	2.42	1.90	1.05	0.55	0.40
0.7	1.88	0.64	0.79	3.88	3.06	1.77	1.03	0.81
0.8	3.41	1.09	1.33	6.82	5.42	3.23	1.99	1.62
0.9	7.99	2.42	2.97	15.56	12.44	7.60	4.82	4.00

Notes: This table contains information relating to the distribution of the performance fee δ^* obtained using the Bayesian approach. Specifically, the mean (MN), standard deviation (SD), interquartile range (IR), and 1, 5, 50, 95, and 99 percentiles are provided. The performance fees are based on a comparison of the $1/N$ rollover strategy (with a window size of eight), and a single-day rollover strategy in which the rollover takes place eights days before the FND.

Table 5 – Performance fees (robustness check)

	Statistic							
Preference $(1-\theta)$	MN	SD	IQR	1%	5%	50%	95%	99%
(a) Early Trading Session (8.30am to 10.00am)								
0.0 (mean-focused)	-0.10	0.16	0.20	0.30	0.16	-0.10	-0.36	-0.50
0.1	0.00	0.17	0.21	0.50	0.30	-0.01	-0.26	-0.37
0.2	0.13	0.22	0.25	0.82	0.51	0.10	-0.16	-0.26
0.3	0.29	0.29	0.31	1.30	0.81	0.24	-0.05	-0.15
0.4	0.51	0.42	0.42	1.99	1.25	0.43	0.06	-0.05
0.5	0.82	0.60	0.58	2.96	1.89	0.69	0.21	0.08
0.6	1.29	0.88	0.84	4.51	2.85	1.08	0.41	0.24
0.7	2.06	1.34	1.29	6.95	4.45	1.73	0.73	0.48
0.8	3.61	2.29	2.20	12.00	7.71	3.04	1.35	0.95
0.9	8.21	5.15	4.88	26.97	17.31	6.96	3.19	2.29
1.0 (variance-focused)	∞							
(b) Mid Trading Session (10.01am to 12.00pm)								
0.0	-0.14	0.16	0.21	0.26	0.12	-0.14	-0.41	-0.54
0.1	-0.04	0.18	0.21	0.46	0.26	-0.05	-0.30	-0.41
0.2	0.09	0.22	0.25	0.78	0.47	0.06	-0.20	-0.30
0.3	0.25	0.30	0.31	1.28	0.77	0.20	-0.09	-0.19
0.4	0.47	0.42	0.42	1.95	1.21	0.39	0.03	-0.08
0.5	0.78	0.59	0.58	2.95	1.85	0.65	0.17	0.04
0.6	1.24	0.87	0.84	4.39	2.79	1.04	0.37	0.20
0.7	2.02	1.34	1.29	7.00	4.39	1.69	0.68	0.44
0.8	3.56	2.28	2.19	11.85	7.66	2.99	1.30	0.90
0.9	8.21	5.16	4.95	27.21	17.44	6.95	3.14	2.28
(c) Late Trading Session (12.01pm to 1.15pm)								
0.0	-0.10	0.15	0.19	0.27	0.15	-0.10	-0.35	-0.48
0.1	0.00	0.17	0.20	0.46	0.28	-0.02	-0.25	-0.36
0.2	0.12	0.21	0.24	0.78	0.48	0.09	-0.16	-0.26
0.3	0.27	0.28	0.30	1.24	0.77	0.22	-0.07	-0.17
0.4	0.47	0.41	0.41	1.90	1.18	0.39	0.03	-0.09
0.5	0.76	0.58	0.57	2.85	1.79	0.63	0.15	0.01
0.6	1.18	0.85	0.83	4.25	2.72	0.98	0.31	0.13
0.7	1.89	1.32	1.26	6.64	4.20	1.58	0.58	0.30
0.8	3.34	2.24	2.16	11.61	7.37	2.79	1.10	0.64
0.9	7.58	5.01	4.77	26.04	16.54	6.38	2.59	1.58

Notes: This table contains information relating to the distribution of the performance fee δ^* obtained using the Bayesian approach applied to corn futures data observed at various points within the trading day. Specifically, the mean (MN), standard deviation (SD), interquartile range (IR), and 1, 5, 50, 95, and 99 percentiles are provided. The performance fees are based on a comparison of the $1/N$ rollover strategy (and a window size of eight), and a single-day rollover strategy in which the rollover takes place eight days before the FND.

(a) Measure 1 (b) Measure 2

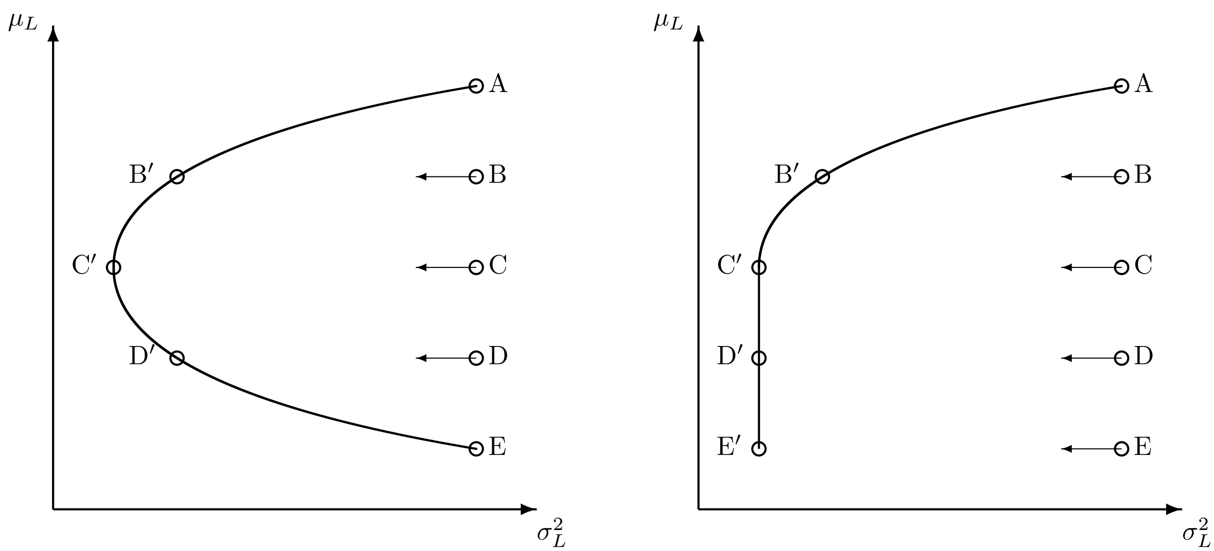

Figure 1 – Performance measurement

This figure contains a graphical representation of two different ways of calculating the variance reductions (for a fixed mean level) available to users of the MVE rollover strategy.

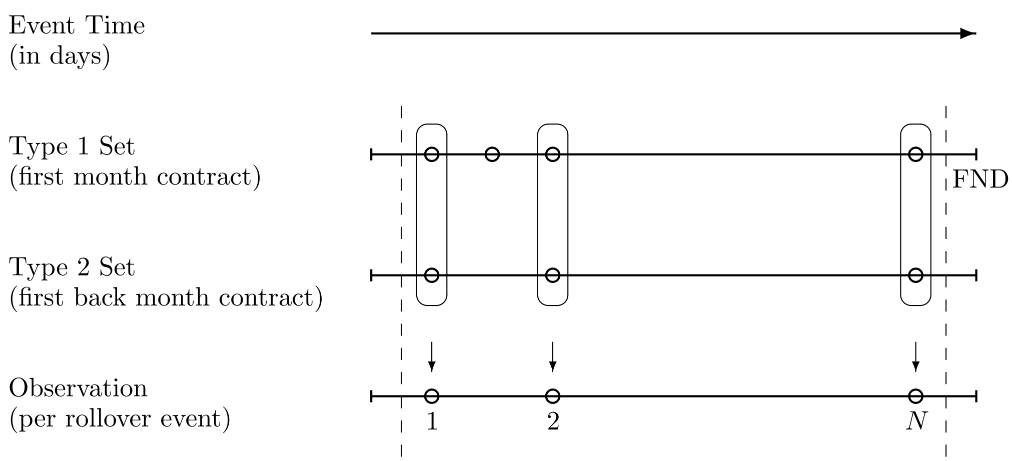

Figure 2 – Dataset construction
This figure contains a visual description of how the dataset is constructed for each rollover event. Each circle represents a data observation within this event.

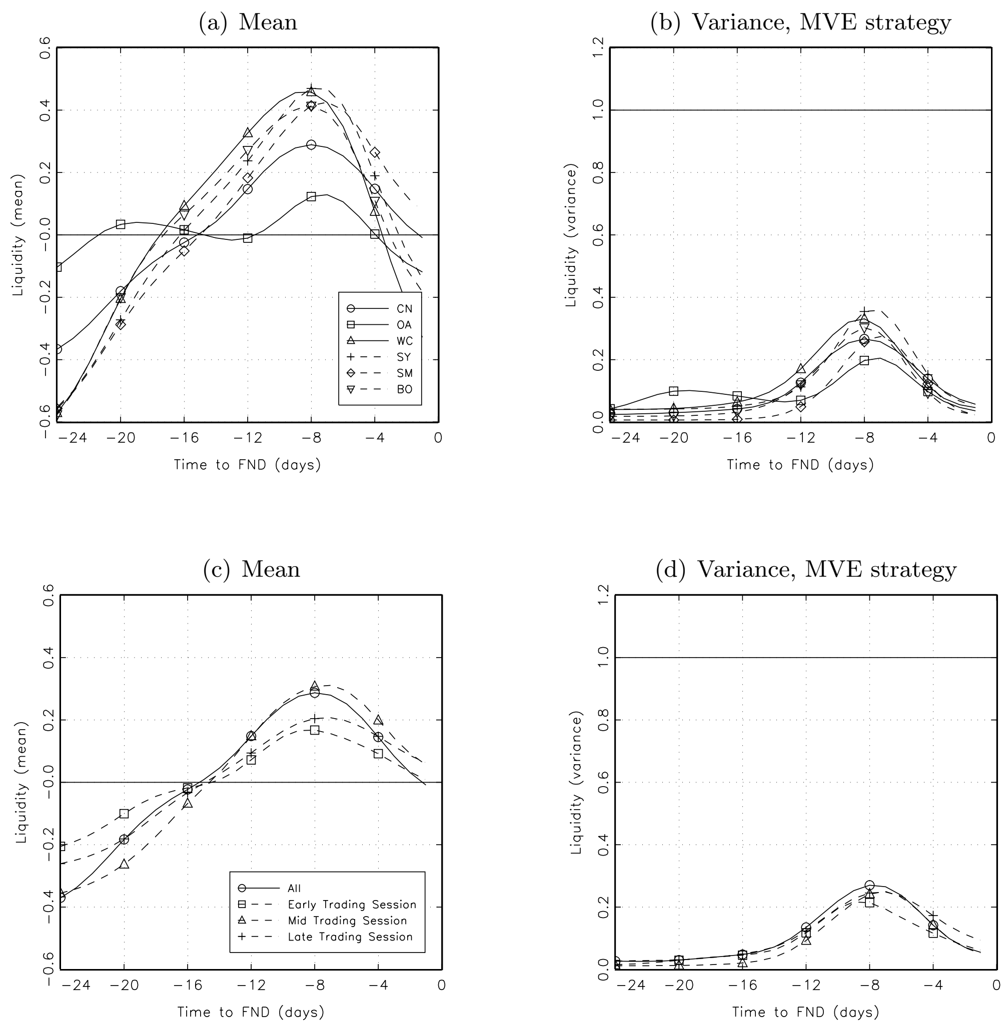

Figure 3 – Liquidity
This figure contains the mean and variance of liquidity associated with all s-day rollover strategies during the 24-day window prior to the FND. Panels (a) and (b) provide information based on for all grain futures observed over all trading periods, while panels (c) and (d) provide information based on corn futures observed during various periods with each trading day.

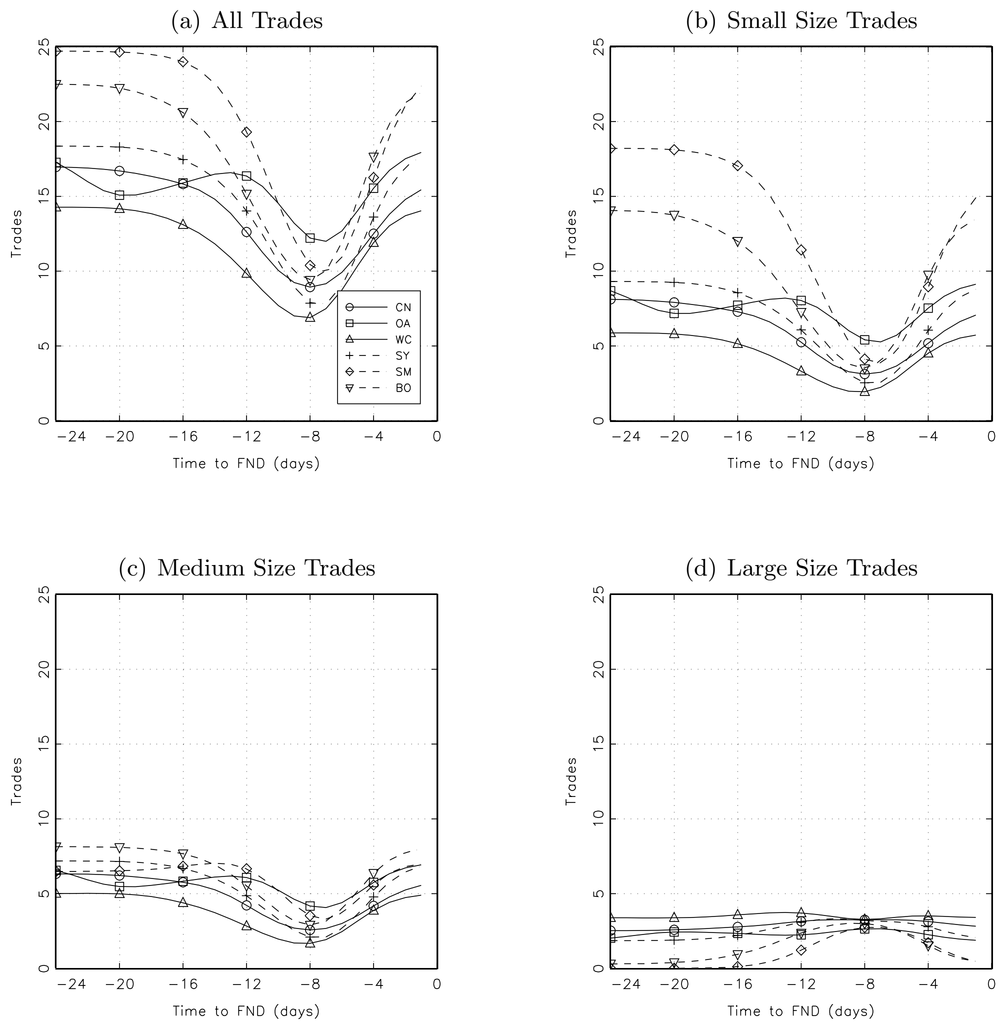

Figure 4 – Strategy characteristics
This figure contains plots of various positions associated with the MVE strategy in terms of the number of non-zero weights (trades on different days in which rollover occurs) required to match the mean aggregate liquidity on each day. Panel (a) contains the mean number of trades, panel (b) contains the mean number of small trades (defined as weights below 0.05), panel (c) contains the mean number of medium trades (defined as weights above 0.05 and below 0.10), and panel (d) contains the mean number of large trades (defined as weights above 0.10).